

C.U.SHAH UNIVERSITY

Winter Examination-2022

Subject Name: Engineering Mathematics-I

Subject Code: 4TE01EMT3

Branch: B.Tech (All)

Semester: 1

Date: 09/01/2023

Time: 11:00 To 02:00

Marks: 70

Instructions:

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

Q-1

Attempt the following questions:

(14)

- a)** The principal argument of $z = 1 + i$. (01)
- (a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{5}$ (d) $\frac{\pi}{6}$
- (b)** The conjugate of $z = \frac{5-3i}{4+3i}$ is _____. (01)
- (a) $\frac{11}{5} + \frac{27}{5}i$ (b) $\frac{11}{5} - \frac{27}{5}i$ (c) $\frac{27}{5} - \frac{11}{5}i$ (d) $\frac{27}{5} + \frac{11}{5}i$
- (c)** The Principal value of $\log(2 + 2i)$ is ? (01)
- (a) $\log 2 + i\frac{\pi}{4}$ (b) $\log 2 - i\frac{\pi}{4}$ (c) $-\frac{1}{2}\log 2 + i\frac{\pi}{4}$ (d) $\frac{1}{2}\log 2 + i\frac{\pi}{4}$
- (d)** The of Eigen values of matrix $A = \begin{bmatrix} 1 & 2 \\ 0 & 4 \end{bmatrix}$ are _____. (01)
- (a) 1,3 (b) 2,3 (c) 1,4 (d) 2,4
- (e)** If $y = \sin(ax + b)$ then $y_n =$ _____. (01)
- (a) $b^2 \sin(ax + b + n\frac{\pi}{2})$ (b) $a^2 \cos(ax + b + n\frac{\pi}{2})$
- (c) $a^n \sin(ax + b + n\frac{\pi}{2})$ (d) $a^n \cos(ax + b + n\frac{\pi}{2})$
- (f)** If $y = e^{3x}$ then $y_n =$ _____. (01)
- (a) $3^{n+1}e^{3x}$ (b) $3^{n-1}e^{3x}$ (c) $3^n e^{3x}$ (d) $3^{n+1}e^{4x}$
- (g)** The Maclaurin's series of $\cos x$ is _____. (01)
- (a) $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}$ (b) $1 + \frac{x^2}{2!} - \frac{x^4}{4!} - \frac{x^6}{6!}$ (c) $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}$ (d) $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!}$
- (h)** Find $\lim_{x \rightarrow \frac{\pi}{2}} \sin x$. (01)
- (a) 0 (b) 1 (c) -1 (d) 2
- (i)** Find $\lim_{x \rightarrow 1} \frac{x^2 - 2x + 1}{2x^2 - 5x + 4}$. (01)
- (a) 0 (b) $-\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $-\frac{1}{3}$
- (j)** The value of $\lim_{\substack{x \rightarrow 1 \\ y \rightarrow 3}} \frac{x^3 + y^3}{x^2 + y^2}$ is _____. (01)



- (a) $\frac{14}{5}$ (b) $\frac{5}{14}$ (c) 5 (d) 14
- (k) The degree of Homogeneous equation $f(x, y) = x^3 + x^2y + y^3$ is_____. (01)
 (a) 2 (b) 3 (c) -2 (d) -3
- (l) The value of $\lim_{x \rightarrow 0} \frac{3^x - 2^x}{x}$ is_____. (01)
 (a) $\log \frac{2}{3}$ (b) $\log \frac{3}{2}$ (c) $\log \frac{9}{4}$ (d) $\log \frac{4}{9}$
- (m) For which value k vectors $v_1 = (1, k), v_2 = (2k, 2)$, are linearly dependent. (01)
 (a) $k = 4$ (b) $k = 3$ (c) $k = 1$ (d) $k = 5$
- (n) The rank of matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 4 \\ 1 & 4 & 5 \end{bmatrix}$ is_____. (01)
 (a) 3 (b) 2 (c) 1 (d) 0

Attempt any four questions from Q-2 to Q-8

Q-2

Attempt all questions

(14)

- (a) If $y = e^{(a \sin^{-1} x)}$ $x \in (-1, 1)$ then prove that $(1 - x^2)y_2 - xy_1 - a^2y = 0$. (05)
 If $y = \cos(ax + b)$ a, b are constant with $a \neq 0$ then prove that (05)
- (b) $y_n = \left\{ \cos \left(ax + b + n \left(\frac{\pi}{2} \right) \right) \right\} n \in N$.
- (c) If $y = \frac{1}{(2x + 1)(3x + 1)}$, $x \neq -\frac{1}{3}, -\frac{1}{2}$ then find y_n . (04)

Q-3

Attempt all questions

(14)

- (a) Find a Maclaurin's series for $f(x) = \cos x$. (05)
 (b) Expand $f(x) = \log x$ in power of $(x - 2)$. (05)
 (c) Expand $f(x) = x^4 - 11x^3 + 43x^2 - 6x + 14$ ascending power of $(x - 1)$. (04)

Q-4

Attempt all questions

(14)

- (a) Find $\lim_{x \rightarrow y} \frac{x^y - y^x}{x^x - y^y}$, where y is constant. (06)
- (b) Find $\lim_{x \rightarrow 0} \frac{\tan x - x}{x^3}$. (04)
- (c) Evaluate $\lim_{x \rightarrow 0} \frac{a^x + b^x + c^x}{3}$. (04)

Q-5

Attempt all questions

(14)

- (a) Find the roots of the equation $z^5 = 1$. (05)
 Prove that
- (b) $(1 + \cos \theta + i \sin \theta)^n + (1 + \cos \theta - i \sin \theta)^n = 2^{n+1} \cos^n \frac{\theta}{2} \cos \left(\frac{n\theta}{2} \right)$. (05)
- (c) Express $\frac{(1 + i)(1 - i)}{(4 - 3i)(4 + 3i)}$ in the form of $a + ib$. (04)

Q-6

Attempt all questions

(14)

- (a) Verify Caley-Hamilton theorem for matrix $A = \begin{bmatrix} 0 & 1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ (06)
- (b) Find the Eigen Values of the matrix $A = \begin{bmatrix} 1 & 0 & 3 \\ 1 & 4 & 5 \\ 1 & 4 & 6 \end{bmatrix}$ (05)



(c) Find the rank of matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ (03)

Q-7

Attempt all questions

(14)

(a) Find the value of $\lim_{\substack{x \rightarrow 0 \\ y \rightarrow 0}} \frac{x^3 - y^3}{x^2 + y^2}$, $x \neq 0, y \neq 0$ (05)

(b) If $u = e^{xyz}$ then prove that $\frac{\partial^3 u}{\partial x \partial y \partial z} = (1 + 3xyz + x^2 y^2 z^2) e^{xyz}$. (05)

(c) If $u = \tan^{-1} \frac{y}{x}$ then find $\frac{\partial^2 u}{\partial x \partial y}$. (04)

Q-8

Attempt all questions

(14)

(a) Express $\cos 8\theta$ in terms of $\cos \theta$ and $\sin \theta$. (06)

(b) Solve system of linear equations by using Gauss-elimination method (05)

$$3x + y - z = 3, x - 2y + 9z = 8, 2x - 8y + z = -5.$$

(c) Evaluate $\lim_{x \rightarrow 0} \frac{e^x + e^{-x} - x^2 - 2}{\sin^2 x - x^2}$. (03)

